Reasoning about Stability

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MVD, October 2014

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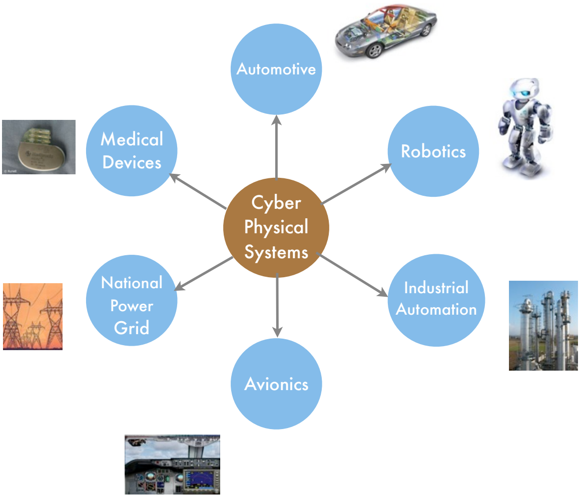
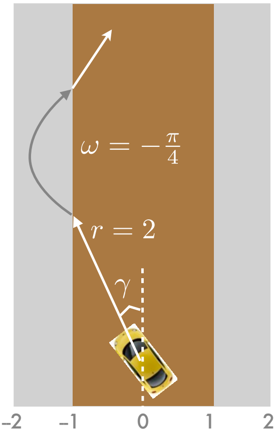
Cyber Physical Systems



Computing devices controlling physical processes arise everywhere

Hybrid Automata



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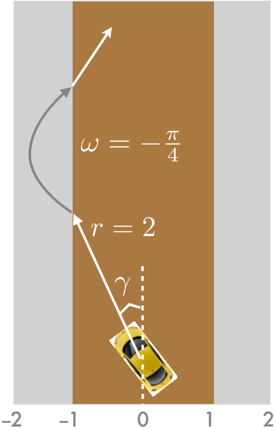

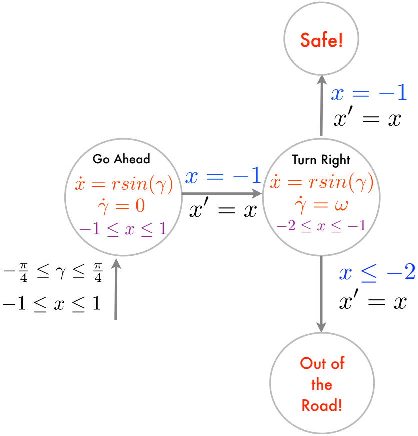

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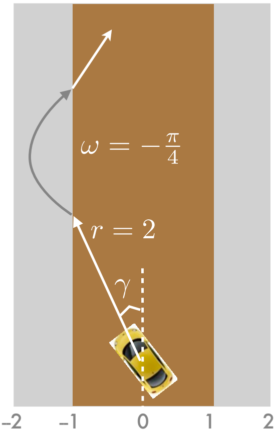

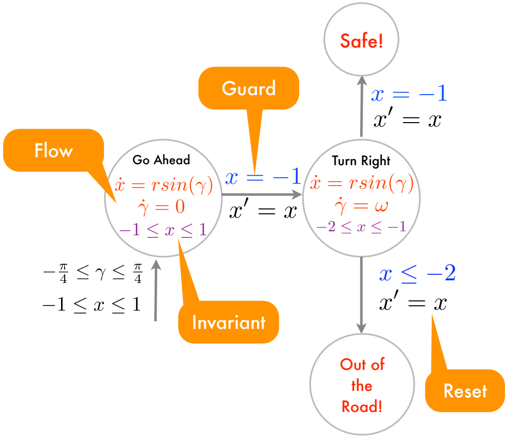

Equilibria in a Pendulum

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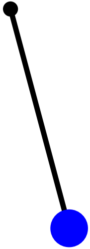

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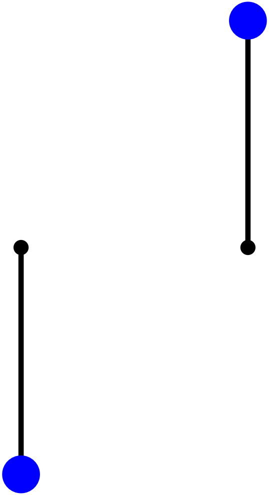

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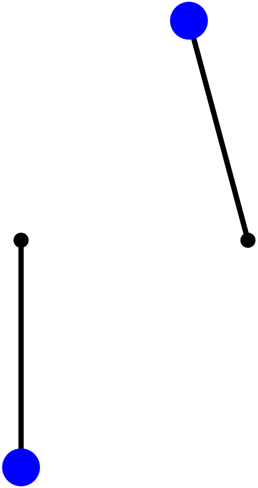

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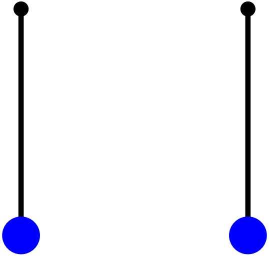
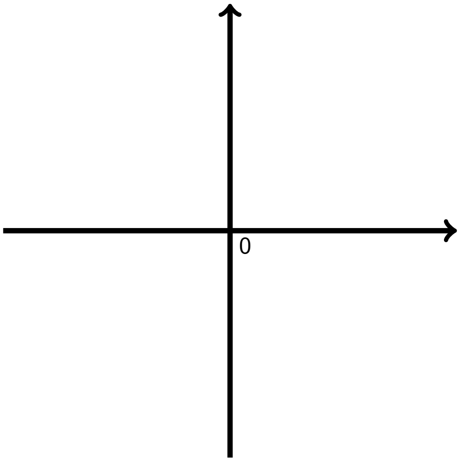


Lyapunov Stability [Aleksandr Lyapunov 1892]

- An equilibrium point is a state from which no executions leave



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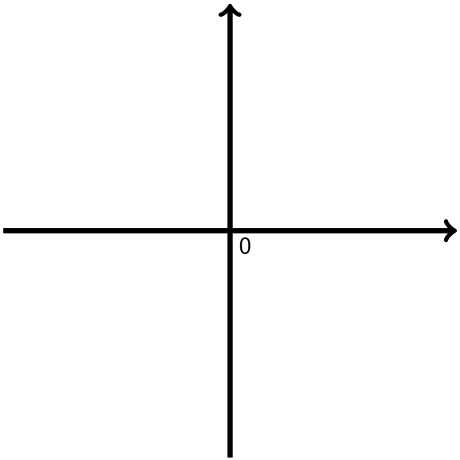

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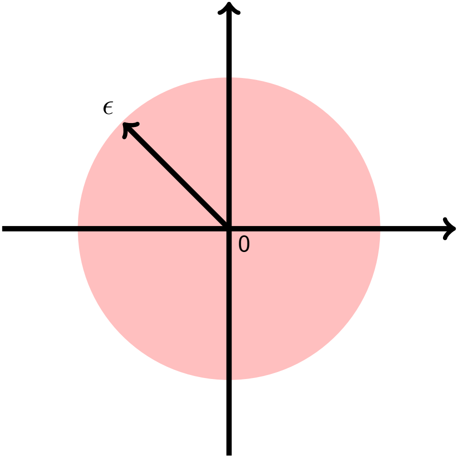

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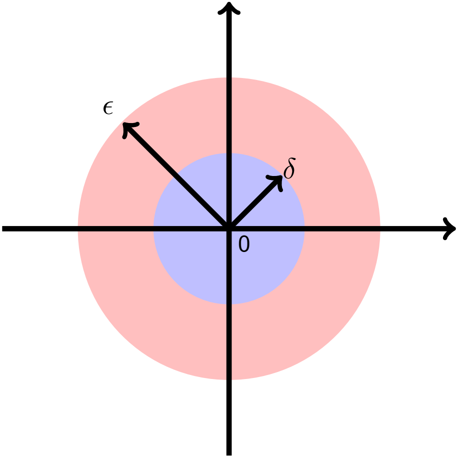
$$\forall \epsilon > 0 \exists \delta > 0.$$



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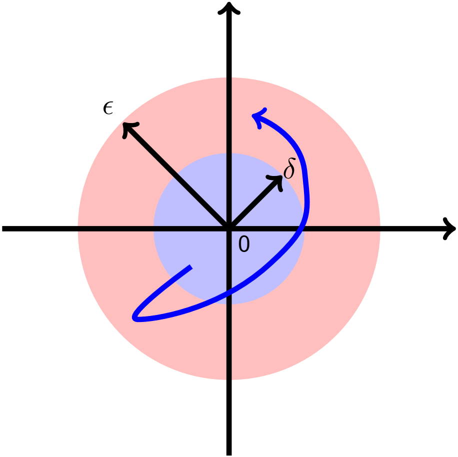

$$\forall \epsilon > 0 \exists \delta > 0.\ \forall \sigma.\ (\sigma(0) \in B_\delta(0)) \rightarrow (\forall t.\ \sigma(t) \in B_\epsilon(0))$$

Asymptotic Stability

Lyapunov Stability +
Convergence

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Lyapunov Stability $+$
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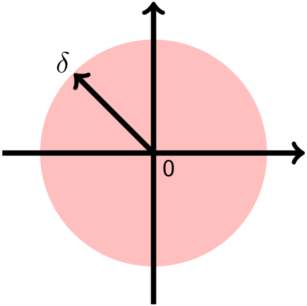

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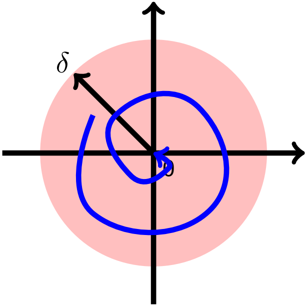

$$\exists \delta > 0 \forall \sigma. \ (\sigma \in B_\delta(0)) \rightarrow \mathrm{Conv}(\sigma)$$

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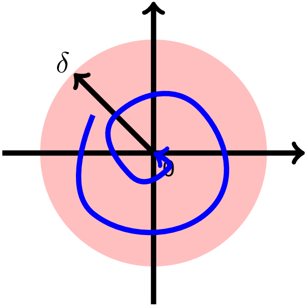

$$\exists \delta > 0 \forall \sigma.\ (\sigma \in B_\delta(0)) \rightarrow \mathrm{Conv}(\sigma)$$
$$\mathrm{Conv}(\sigma) \equiv \forall \epsilon > 0. \exists T. \forall t \geq T.\ \sigma(t) \in B_\epsilon(0)$$

Talk Outline

- Why is the problem of checking stability different?

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- How difficult is checking stability computationally?

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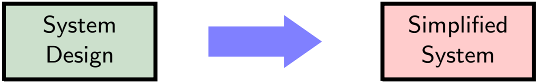
- Why is the problem of checking stability different?
- How difficult is checking stability computationally?
- What are proof principles to check stability?

Part I

Why is stability different?

Prabhakar-Dullerud-Viswanathan

Process Preorders and Equivalences



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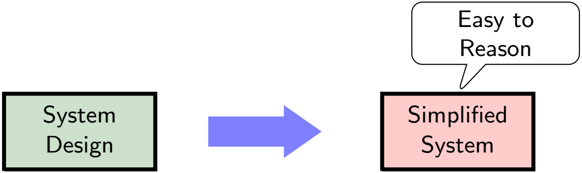

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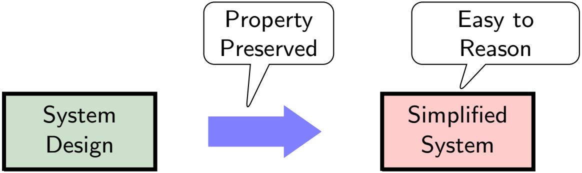

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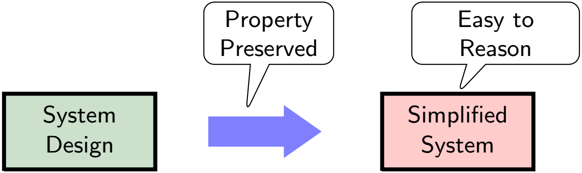

Process preorders and equivalences characterize the relationship between complex and simple systems that preserve properties of interest

Transition Systems

$\mathcal{T} = (S, \Sigma, \longrightarrow)$, where S is a set of states, Σ is a set of actions, and $\longrightarrow \subseteq S \times \Sigma \times S$ is the transition relation.

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Bisimulation

$R \subseteq S_1 \times S_2$ is a bisimulation between $\mathcal{T}_1 = (S_1, \Sigma, \longrightarrow_1)$ and $\mathcal{T}_2 = (S_2, \Sigma, \longrightarrow_2)$ iff for every $(p, q) \in R$

Bisimulation

For Labeled Transition Systems

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- If $p \xrightarrow{a}_1 p'$ then there is $q' \in S_2$ s.t. $q \xrightarrow{a}_2 q'$ and $(p', q') \in R$, and

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- If $q \xrightarrow{a}_2 q'$ then there is $p' \in S_1$ s.t. $p \xrightarrow{a}_1 p'$ and $(p', q') \in R$.

Hybrid Transition Systems

$\mathcal{T} = (S, \Sigma, \longrightarrow, \Delta)$, where S is a set of states, Σ is a set of actions, $\longrightarrow \subseteq S \times \Sigma \times S$ is the transition relation, and $\Delta \subseteq \{\sigma \mid \sigma : [0, T] \to S\}$ is a set of trajectories.

Bisimulation

For Hybrid Transition Systems

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- If $\tau_1 \in \Delta_(p)$ then there is $\tau_2 \in \Delta_2(q)$ s.t. $\forall t$, $(\tau_1(t), \tau_2(t)) \in R$, and
- "Conversely"

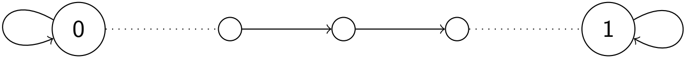
Features of Bisimulation

- The cannonical notion of equivalence between systems
- Preserves all categories of properties like safety, liveness, branching time, etc.
- Basis for minimization and decidability results

Is stability preserved by bisimulation?

Is stability preserved by bisimulation?
No!

Stability not Bisimulation Invariant



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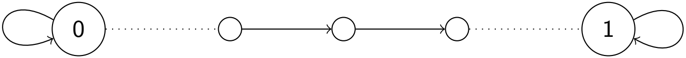

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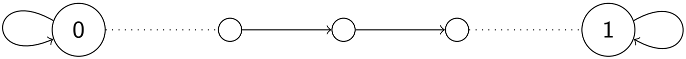

- Dynamics: $\sigma(x_0, t) = x_0^{\frac{1}{2t}}$
- Bisimulation: $R = [0,1] \times [0,1]$ is a bisimulation
- 1 is Lyapunov/Asymptotically stable but 0 is not!

Need continuity!

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Definition

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$$\forall \epsilon > 0 \exists \delta > 0 \forall x \in \mathrm{dom}(R).\ R(B_\delta(x)) \subseteq B_\epsilon(R(x))$$

Invariance under Uniformly Continuous Bisimulations

Theorem

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be hybrid transition systems with 0 as an equilibrium point. Suppose R is a uniformly continuous bisimulation such that $(0,0) \in R$ then

$\mathcal{T}_1$ is Lyapunov stable w.r.t. 0 iff $\mathcal{T}_2$ is Lyapunov stable w.r.t. 0

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- Above observation generalizes to stronger notions of stability

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- Above observation generalizes to stronger notions of stability
- Uniformly continuous simulations reflect stability notions

Lyapunov's Second Method

System $\dot{x} = F(x)$ with solution $\varphi(x, t)$

Equilibrium $F(0) = 0$

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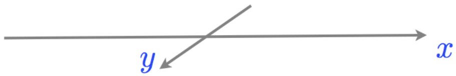
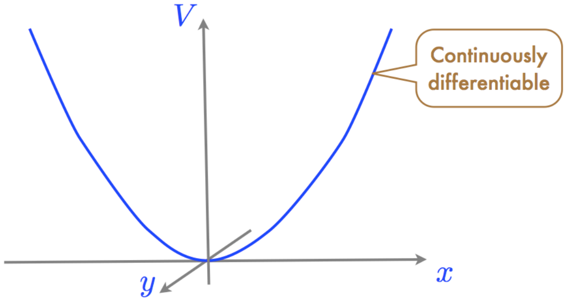
If there exists a "Lyapunov function" for the system then it is Lyapunov stable.


y
x

Lyapunov Function

An Illustration



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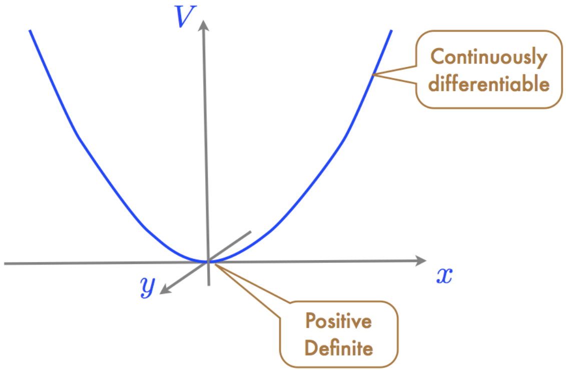

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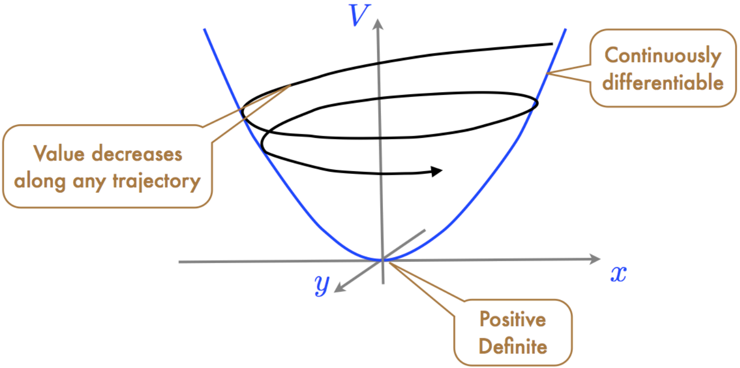

Lyapunov's Method as an Abstraction

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V is positive definite C^1
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Lyapunov's Method as an Abstraction

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$v_1 \xrightarrow{t} v_2$ iff exist x_1, x_2 s.t. $x_1 \xrightarrow{t} x_2$, $V(x_1) = v_1$ and $V(x_2) = v_2$.

Lyapunov's Method as an Abstraction

Easily Stable

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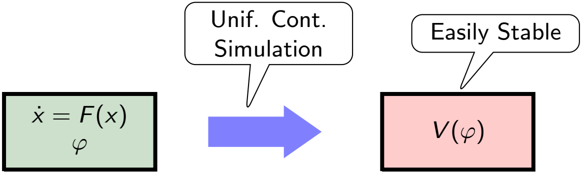
Lyapunov's Method as an Abstraction



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Other Characterizations

- Other extensions of Lyapunov's method to switched systems can also be understood in the abstraction setting
- Hartman-Grobman Theorem contructs a uniformly continuous bisimilar linearization

Expressing Stability

- Stabiity cannot be expressed in the classical modal/temporal logics like Hennessy-Milner, LTL, CTL, μ-calculus, etc.
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- Stabiity cannot be expressed in the classical modal/temporal logics like Hennessy-Milner, LTL, CTL, μ-calculus, etc.
  - Logic equivalence for these logics coincides with bisimulation
- First order logic over appropriate topologial structures is too strong
- Is there a (modal) logic that can express stability for which logic equivalence coincides with "continuous" bisimulations?

S_4: A Modal Logic for Space

Orlov[1928], Lewis[1932], Gödel[1933], Stone[1937], Tarski[1937]

$$\varphi ::= p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \mathbf{I}\varphi \mid \mathbf{C}\varphi$$

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- Formulas interpreted as sets of points in a topological space
- $[\![\mathbf{I}\varphi]\!]$: Interior of set defined by φ
- $[\![\mathbf{C}\varphi]\!]$: Closure of set defined by φ

Bimodal Spatio-Temporal Logic

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Bimodal Spatio-Temporal Logic

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- Can stability be expressed in the resulting logic? No!
  - [Aiello-van Bentham, Davoren] Logic equivalence in this bi-modal logic coincides with bisimilarity under relations with "weak continuity" properties
- Open Question: What is the right logic?

Part II

How difficult is checking stability computationally?

Prabhakar-Viswanathan

What is known about checking Stability?

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- Traditional control theoretic methods focus on identifying sufficient conditions that guarantee stability
  - Some complexity results on how difficult it is to find these sufficient conditions
- [Blondel-Tsitsiklis et. al.] Prove computational lower bounds (undecidability/NP-hardness) on checking stability of special discrete time linear switched systems

Deciding Hybrid Models

Traditional techniques to establish decidablity for various properties (like safety, liveness, etc.) of special hybrid models fail for stability

Establishing stability needs new ideas

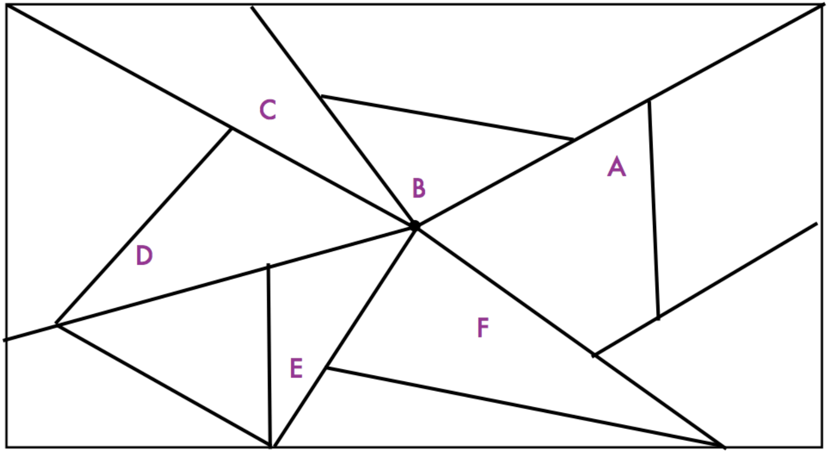
Deciding Hybrid Models

Traditional techniques to establish decidablity for various properties (like safety, liveness, etc.) of special hybrid models fail for stability

- Decidability results rely on establishing the existence of an effectively constructable finite "bisimulation" quotient

Establishing stability needs new ideas

Piecewise Constant Derivative (PCD) Systems



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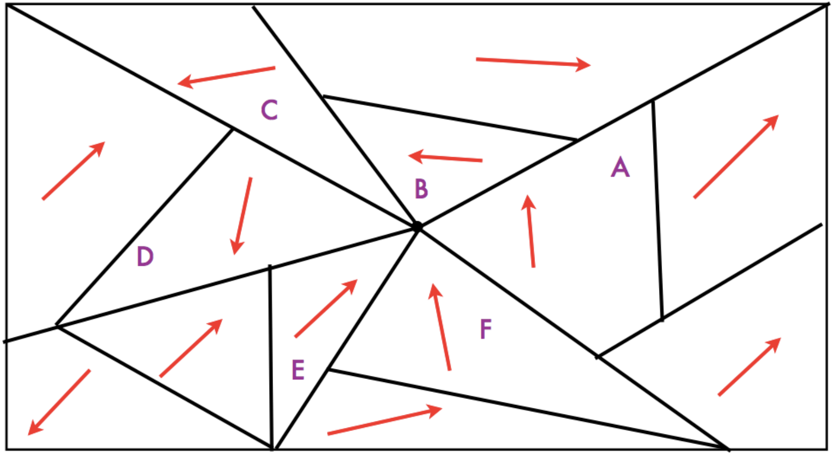

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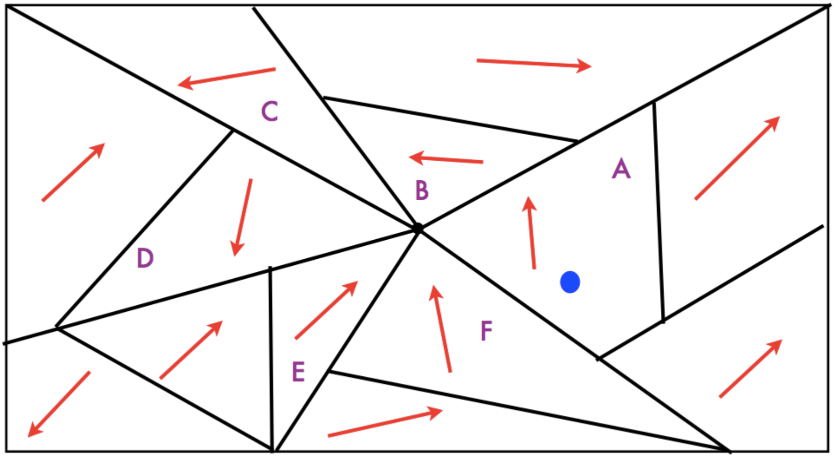

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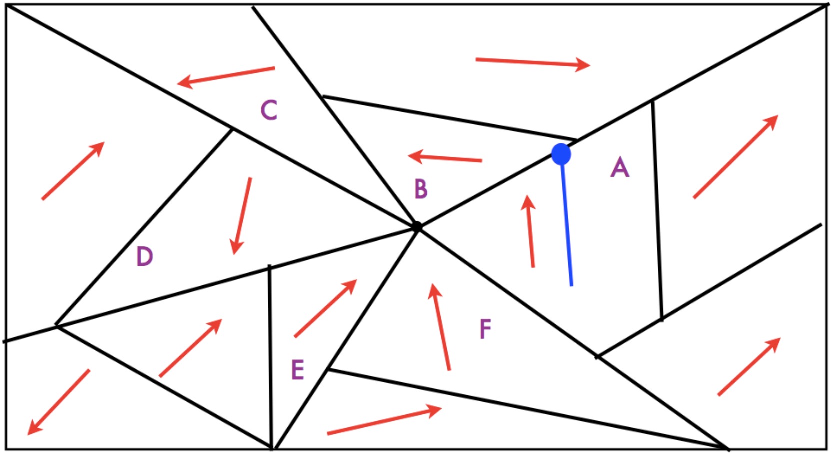

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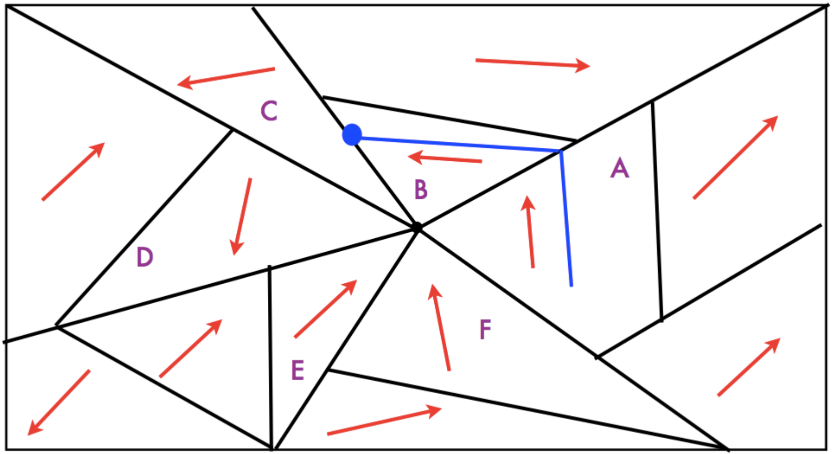

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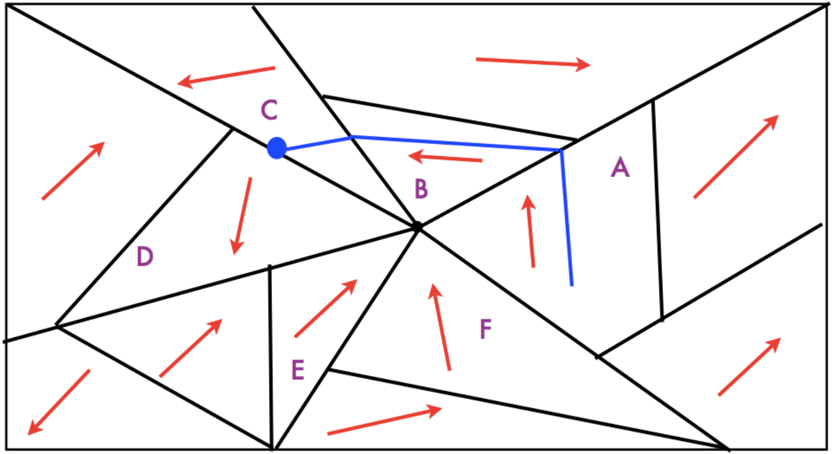

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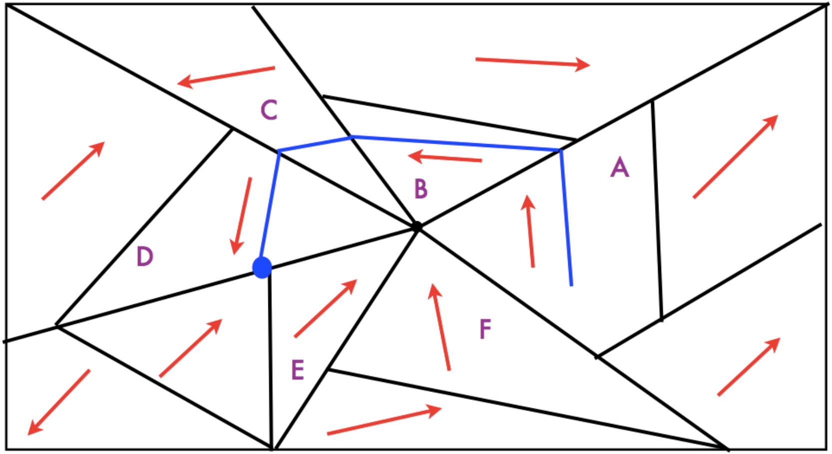
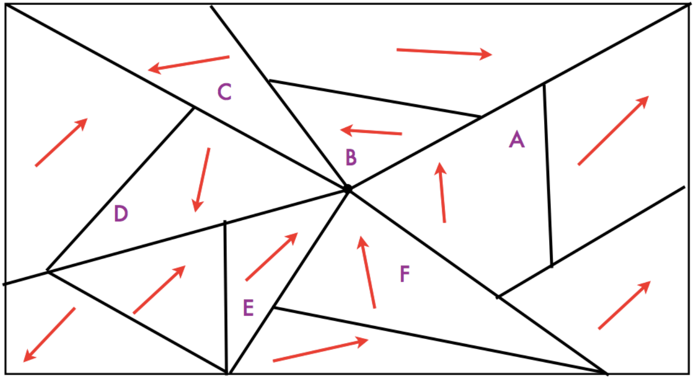
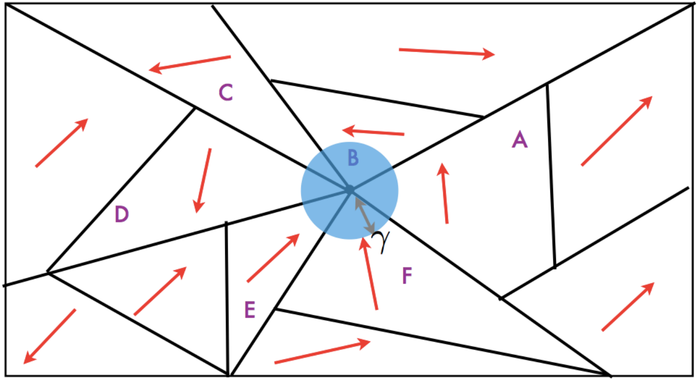


Stability in PCD



$$\forall \epsilon > 0\ \exists \delta > 0\ \forall \sigma.\ (\sigma(0) \in B_\delta(0)) \to (\forall t.\ \sigma(t) \in B_\epsilon(0))$$

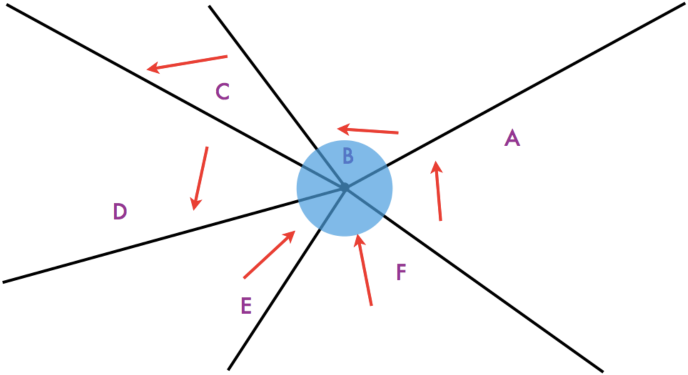
Stability in PCD



$\exists\gamma > 0.\ \forall\epsilon \in (0,\gamma]$

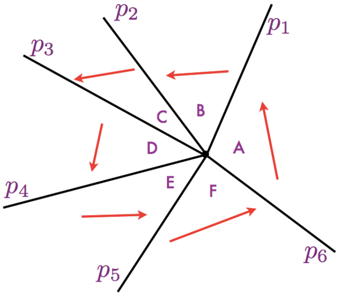
$$\cancel{\forall\epsilon > 0}\ \exists\delta > 0\ \forall\sigma.\ (\sigma(0) \in B_\delta(0)) \rightarrow (\forall t.\ \sigma(t) \in B_\epsilon(0))$$

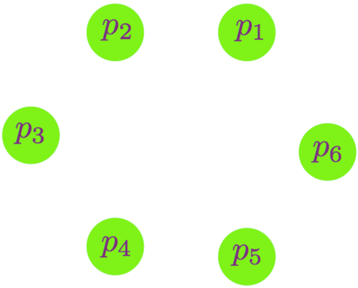
Special Structure Near the Origin



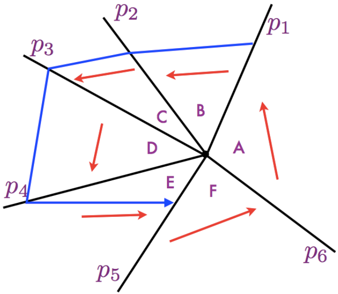
The planar partition looks like "wedges".

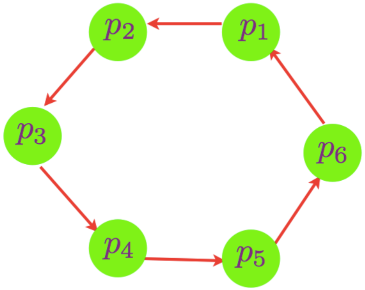
Executions near 0



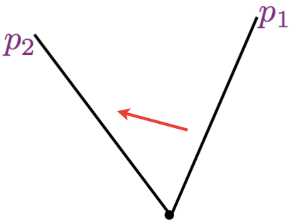


Executions near 0





Capturing distance from origin



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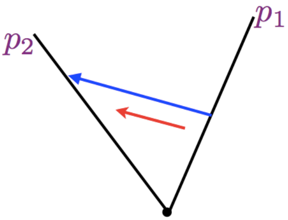

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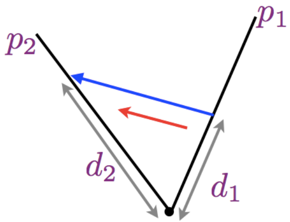
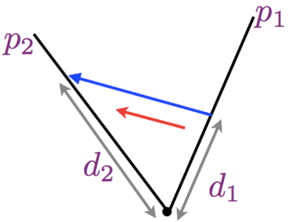
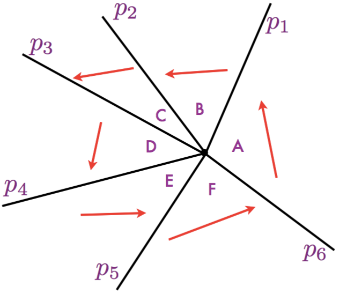


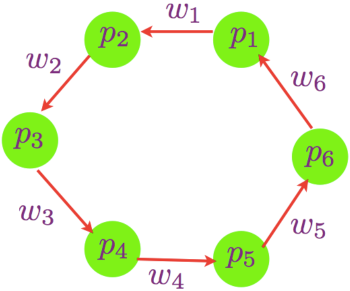
Capturing distance from origin

$$w(e) = \frac{|d_1|}{|d_2|}$$



Stability Analysis
Weighted Graph





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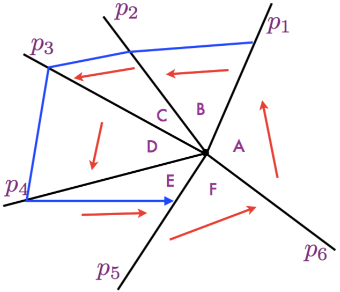
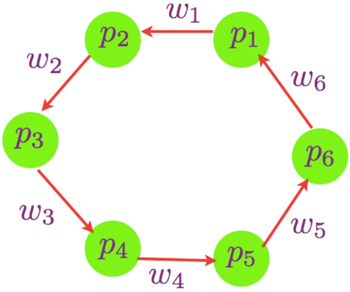
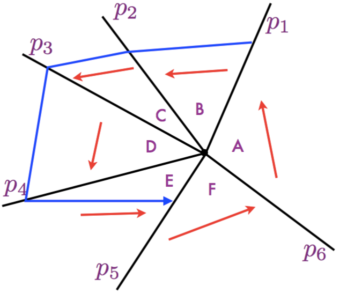
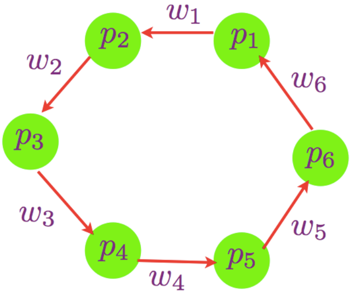




$$\sigma = \sigma(p_1, p_2)\sigma(p_2, p_3)\cdots\sigma(p_4, p_5)$$

Stability Analysis
Weighted Graph





$$\sigma = \sigma(p_1, p_2)\sigma(p_2, p_3) \cdots \sigma(p_4, p_5)$$
$$w(\sigma) = \frac{|d(\sigma(T))|}{|d(\sigma(0))|}$$

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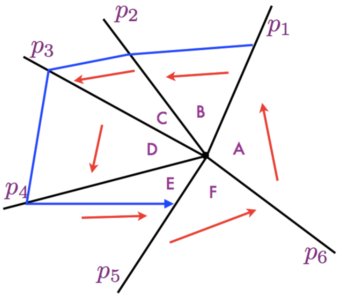
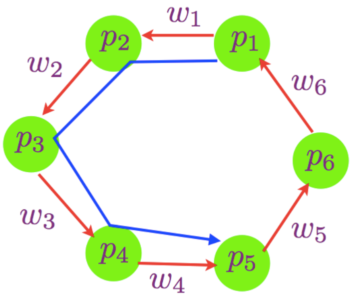
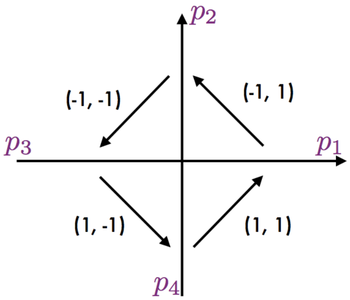
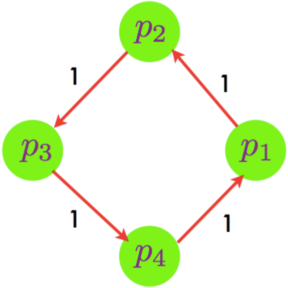




$$\sigma = \sigma(p_1, p_2)\sigma(p_2, p_3)\cdots\sigma(p_4, p_5)$$
$$w(\sigma) = \frac{|d(\sigma(T))|}{|d(\sigma(0))|} = w(p_1, p_2)w(p_2, p_3)\cdots w(p_4, p_5)$$

Example 1





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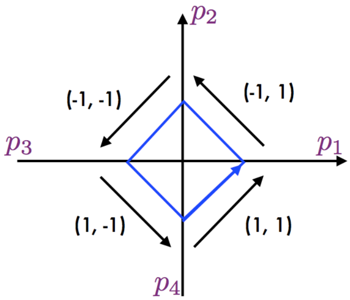

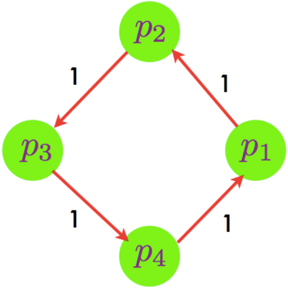

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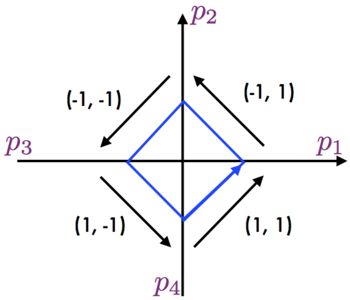
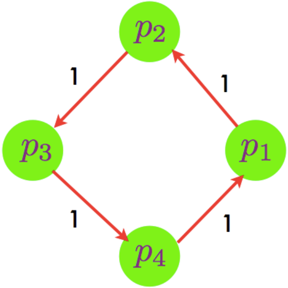
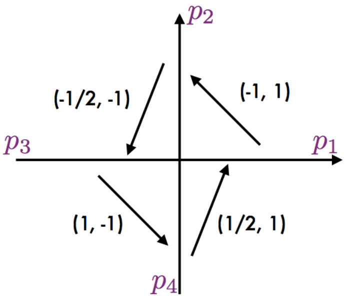
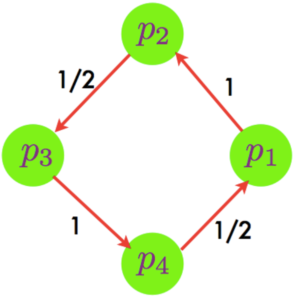




Lyapunov stable but not asymptotically stable.

Example 2





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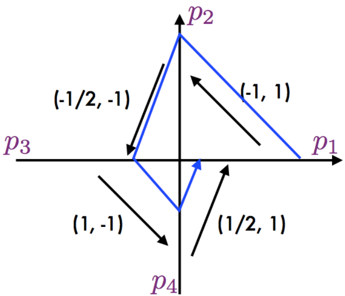

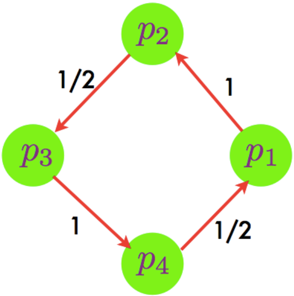

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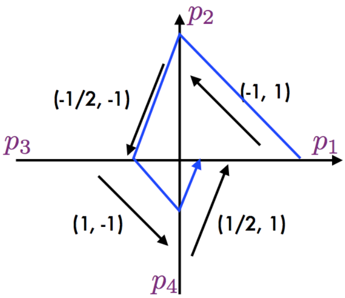
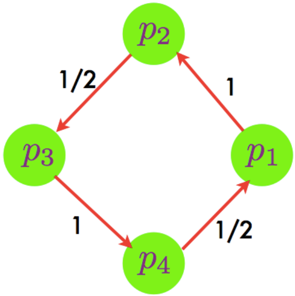
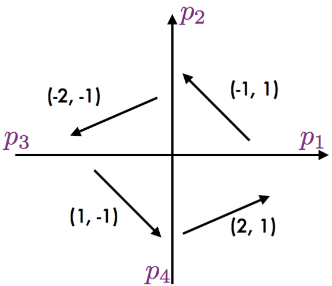
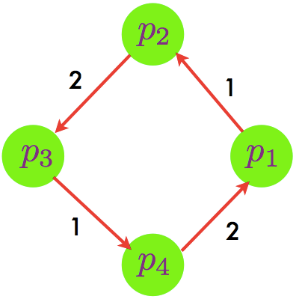




Lyapunov and asymptotically stable.

Example 3





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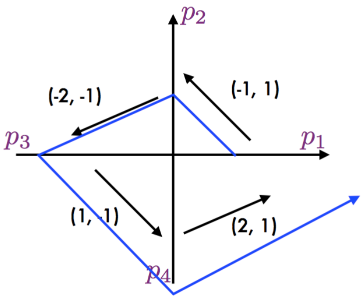

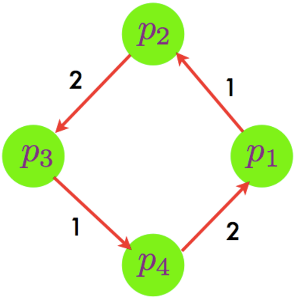

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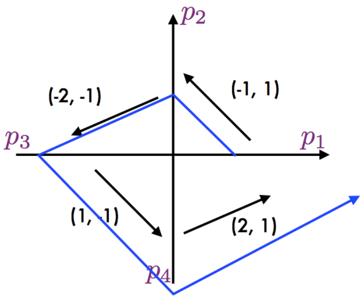

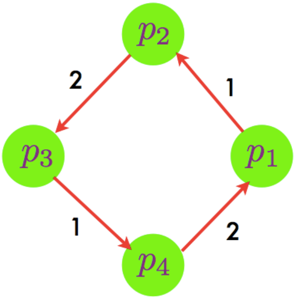

Neither Lyapunov nor asymptotically stable.

Stability of PCD

Theorem

A piecewise constant derivative system is Lyapunov stable iff the weighted graph does not have any cycles of weight > 1 *.*

Stability of PCD

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Decidability Results

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- Stability problem is undecidable for PCD in 5 dimensions.

Stability, Safety, and Abstraction

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- However, reachability is not known to be decidable for planar rectangular switched systems with polyhedral guards and resets; stability algorithm exploits special structure near origin
- Stability algorithm can be thought of as constructing a "quantitative predicate abstraction".Can be exploited to do abstraction-based checking of stabiity of more general hybrid systems.

Proof Rules for reasoning about Stability
[Roohi-Dullerud-Viswanathan]

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Conclusions

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- Concrete abstraction based methods to verify stability
- Logic to specify stability
- Computational complexity of checking stability
- Relationship between Safety and Stability
- Compositional reasoning principles